Supplementary Information

All-silicon low-loss THz temporal differentiator based on whispering gallery mode resonator waveguide platform

Yunjie Rui[1,5], Shuyu Zhou[1,5], Xuecou Tu[1,2,*], Xu Yan[1], Bingnan Yan[1], Chen Zhang[1], Ziyao Ye[1], Huilin Zhang[1], Jingya Xie[4], Qing-Yuan Zhao[1,3], La-Bao Zhang[1,2], Xiao-Qing Jia[1,2], Huabing Wang[1,3], Jian Chen[1,3], Lin Kang[1,2] and Peiheng Wu[1,2]

[1]Research Institute of Superconductor Electronics (RISE), School of Electronic Science and Engineering, Nanjing University, Nanjing, Jiangsu 210023, China

[2]Hefei National Laboratory, Hefei 230088, China

[3]Purple Mountain Laboratories, Nanjing, Jiangsu 211111, China

[4]Terahertz Technology Innovation Research Institute, Shanghai Key Lab of Modern Optical System, University of Shanghai for Science and Technology, Shanghai, 200093, China

[5]These authors contributed equally: Yunjie Rui and Shuyu Zhou

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[*]Corresponding authors. Email: X.T. (tuxuecou@nju.edu.cn)

Supplementary Note 1: First-order differentiator in the time domain (IFOD)

In this section, we will demonstrate that a WGMR waveguide operating in the critical coupling resonance can approximate a time differentiator.

The basic structure of the silicon WGMR waveguide used in this study involves a ring coupled to a single straight waveguide. According to the coupling mode theory, the transfer function of the microring resonator can be expressed as follows:

$$T(\omega) = \frac{s_0}{s_i} = \frac{j(\omega - \omega_0) + \frac{1}{\tau_i} - \frac{1}{\tau_e}}{j(\omega - \omega_0) + \frac{1}{\tau_i} + \frac{1}{\tau_e}} \tag{1}$$

where ω_0 is the resonance frequency, $1/\tau_i$ is the power decay rate due to intrinsic losses, $1/\tau_e$ is the power coupled to the waveguide, and $1/\tau$ is the photon lifetime, with $1/\tau = 1/\tau_i + 1/\tau_e$. When the frequency detuning is much smaller than the 3 dB bandwidth of the resonator, the expression can be approximated as follows:

$$T(\omega) = j\tau(\omega - \omega_0) + \frac{\frac{1}{\tau_i} - \frac{1}{\tau_e}}{\frac{1}{\tau_i} + \frac{1}{\tau_e}} \tag{2}$$

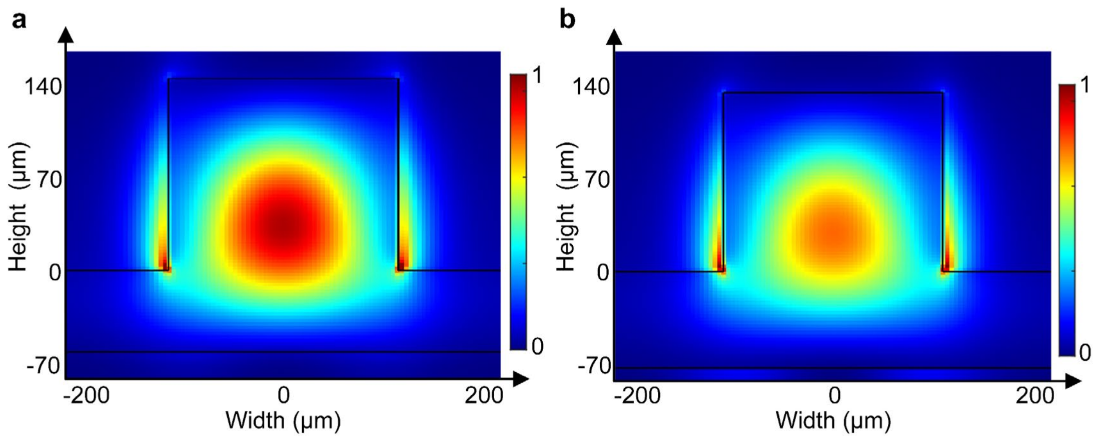
In particular, when the WGMR waveguide operates in the critical coupling resonance ($\tau_i = \tau_e$), the following expression can be obtained:

$$T(\omega) = j\tau(\omega - \omega_0) \tag{3}$$

Equation (3) is essentially consistent with the transfer function of an ideal first-order differentiator in the time domain.

Supplementary Note 2: Waveguide dimension optimization

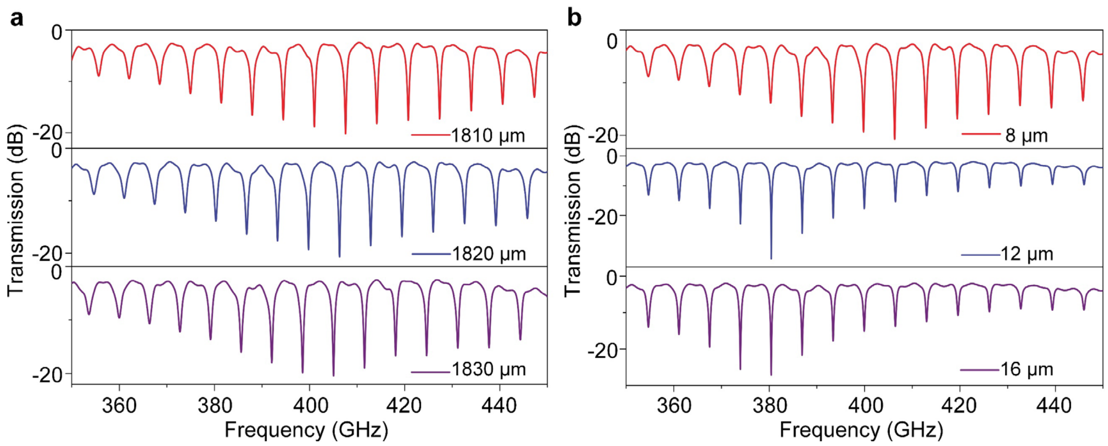
The designed differentiator employs a ridge waveguide. The thickness of the ridge waveguide substrate and the height and width of the waveguide all influence the mode propagation within the waveguide. **Supplementary Figure 1a** illustrates the TE mode energy distribution of the cross-section of a ridge waveguide with a substrate thickness of 60 μm, waveguide height of 140 μm, and waveguide width of 210 μm at a frequency of 405.45 GHz. In this dimension, the electromagnetic wave energy is well confined within the dielectric waveguide, ensuring efficient transmission of the TE mode. When the substrate thickness is changed to 70 μm, and the waveguide height is changed to 130 μm while keeping the waveguide width constant, the calculated TE mode energy distribution of the ridge waveguide cross-section is shown in **Supplementary Figure 1b**. The energy at the center of the dielectric waveguide significantly weakens, indicating increased transmission loss. Hence, the size of the ridge waveguide greatly affects the transmission, necessitating careful design. By utilizing electromagnetic simulation software, the final optimization dimensions of the ridge waveguide used in the original study were determined.



Supplementary Figure 1: Cross-sectional electric field distribution of the waveguide at different dimensions.

In addition to determination of ridge waveguide dimensions, the radius of the microring and the gap between the microring and the straight waveguide also influence the transmission characteristics of the entire resonator. The following optimization will focus on the impact of these parameters from the perspective of the WGMR waveguide's transmission.

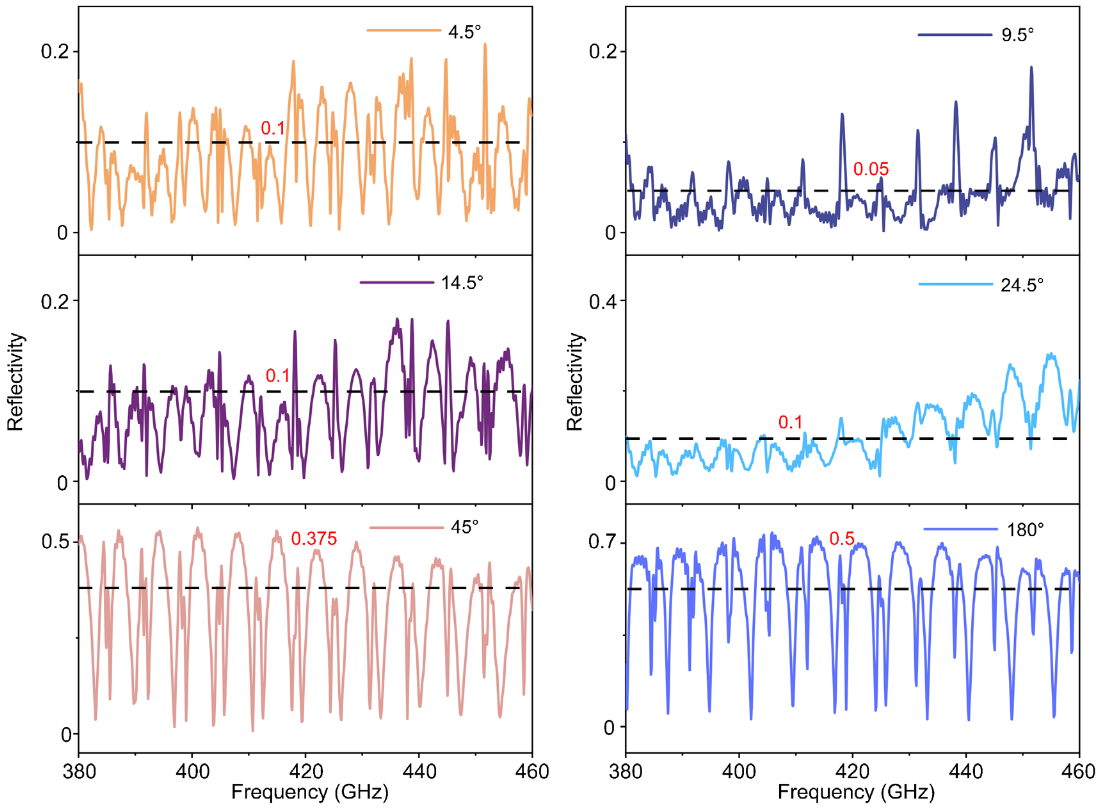
Supplementary Figure 2a shows the calculated transmission of the WGMR waveguide chip obtained at different radii. It can be observed that the radius size affects the position of the critical coupling point. As the radius increases, the critical coupling frequency decreases. **Supplementary Figure 2b** illustrates the calculated transmission of the WGMR waveguide chip at different gaps between the straight waveguide and the microring. It is evident that the gap size not only affects the position of the critical coupling point but also influences the depth of the resonance peak. Hence, these two parameters significantly impact the resonant characteristics of the differentiator, and by continuous optimization, the final dimensions of the radius and gap, as mentioned in the paper, were determined.



Supplementary Figure 2: Calculated transmission at different radii and different gaps.

Supplementary Note 3: Taper angle optimization

In order to mitigate the coupling loss caused by energy entering the dielectric waveguide, a gradient taper structure is employed at both ends of the straight waveguide. To determine the optimal taper angle of the taper, a parameter sweep is conducted on the taper angle parameter (4.5º, 9.5º, 14.5º, 24.5º, 45º, 180º and so on), and the reflectance of the waveguide port is calculated, as shown in **Supplementary Figure 3**. The results indicate that when the taper angle is set to 9.5°, the port reflectance is below 0.05, significantly lower than the reflectance at other angles. In the absence of the taper, the port reflectance exceeds 0.5, highlighting the necessity of incorporating the taper in the WGMR waveguide chip. Furthermore, when the taper angle is set to 9.5°, the calculated insertion loss is only 2.5 dB, as stated in the main text of the paper, demonstrates that this angle yields high coupling efficiency and meets the design requirements.



Supplementary Figure 3: The taper angle effect on coupling efficiency, the calculated reflectivity of the WGMR waveguide chip under different angles of the taper from 4.5° to 180°.